Experiment 5
Pulse Amplitude Modulation (PAM)

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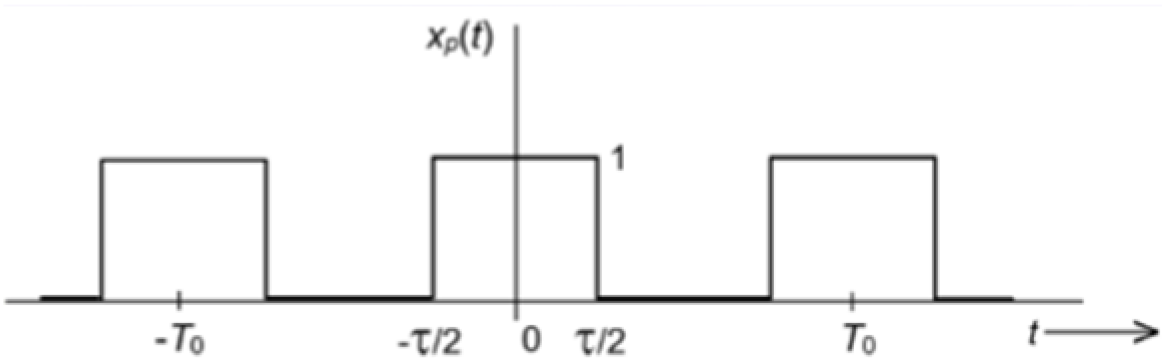
The periodic square function

- In this experiment, we will use the periodic square function to perform the task of sampling a message signal.
- For this signal, we can control both the frequency and the duty cycle, defined as the ratio between the ON time of the signal and the period.
- The periodic square wave is shown in the figure



The Fourier series coefficients of this signal can be found to be

$$x_n = \left(\frac{\tau}{T_0}\right)\mathrm{sinc}(n f_0 \tau)$$

- In the experiment, you will be required to vary the ratio τ/T_0 and find the frequency at which the n'th harmonic becomes zero.
- f_0 is the fundamental frequency

The periodic square function

- The first few terms in the Fourier expansion of x(t) when the ratio τ/T_0 = 1/2 are shown below in the time and frequency domains. For this specific ration of ½, the spectral component at $2f_0$ becomes 0.
- You should observe a similar figure in the frequency domain.

$$\frac{4}{\pi}\left(\sin(2\pi f t) + \frac{1}{3}\sin(2\pi(3f)t) + \frac{1}{5}\sin(2\pi(5f)t) + \frac{1}{7}\sin(2\pi(7f)t)\right)$$

$$\frac{4}{\pi}\sin(2\pi f t)$$

Fundamental Term

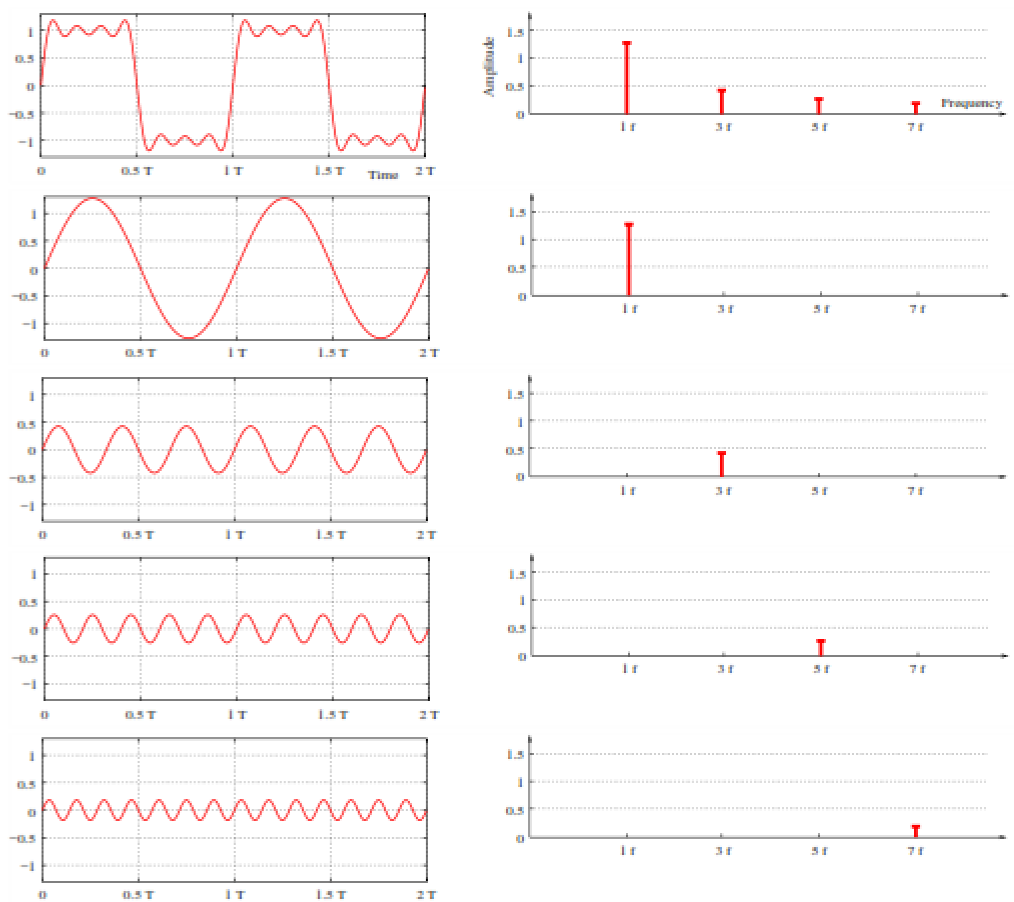
$$\frac{4}{\pi}\times\frac{1}{3}\sin(2\pi(3f)t)$$

Third harmonic

$$\frac{4}{\pi}\times\frac{1}{5}\sin(2\pi(5f)t)$$

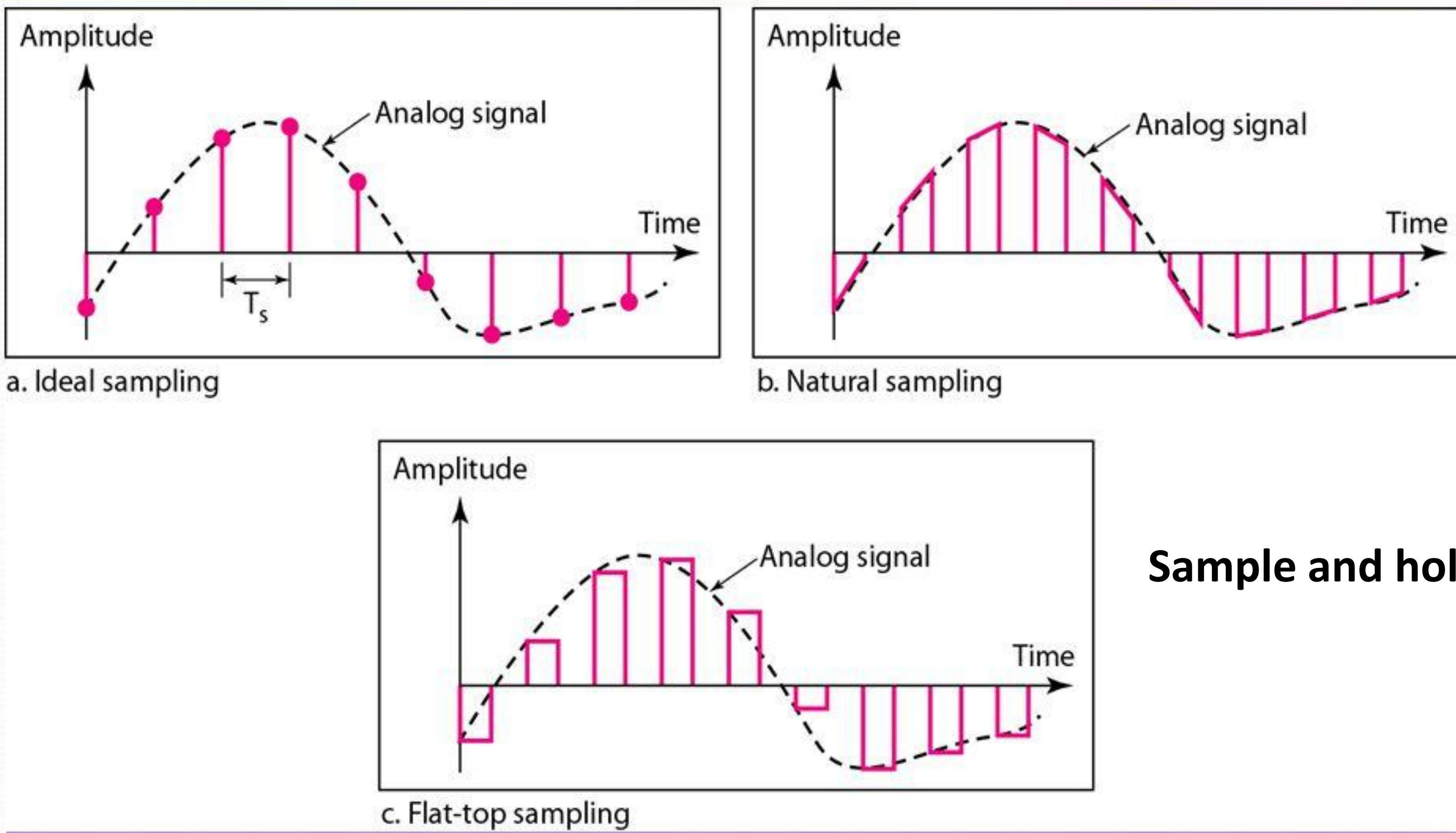
Fifth harmonic

$$\frac{4}{\pi}\times\frac{1}{7}\sin(2\pi(7f)t)$$



Sampling Theorem and Sampling Techniques

Three different sampling methods



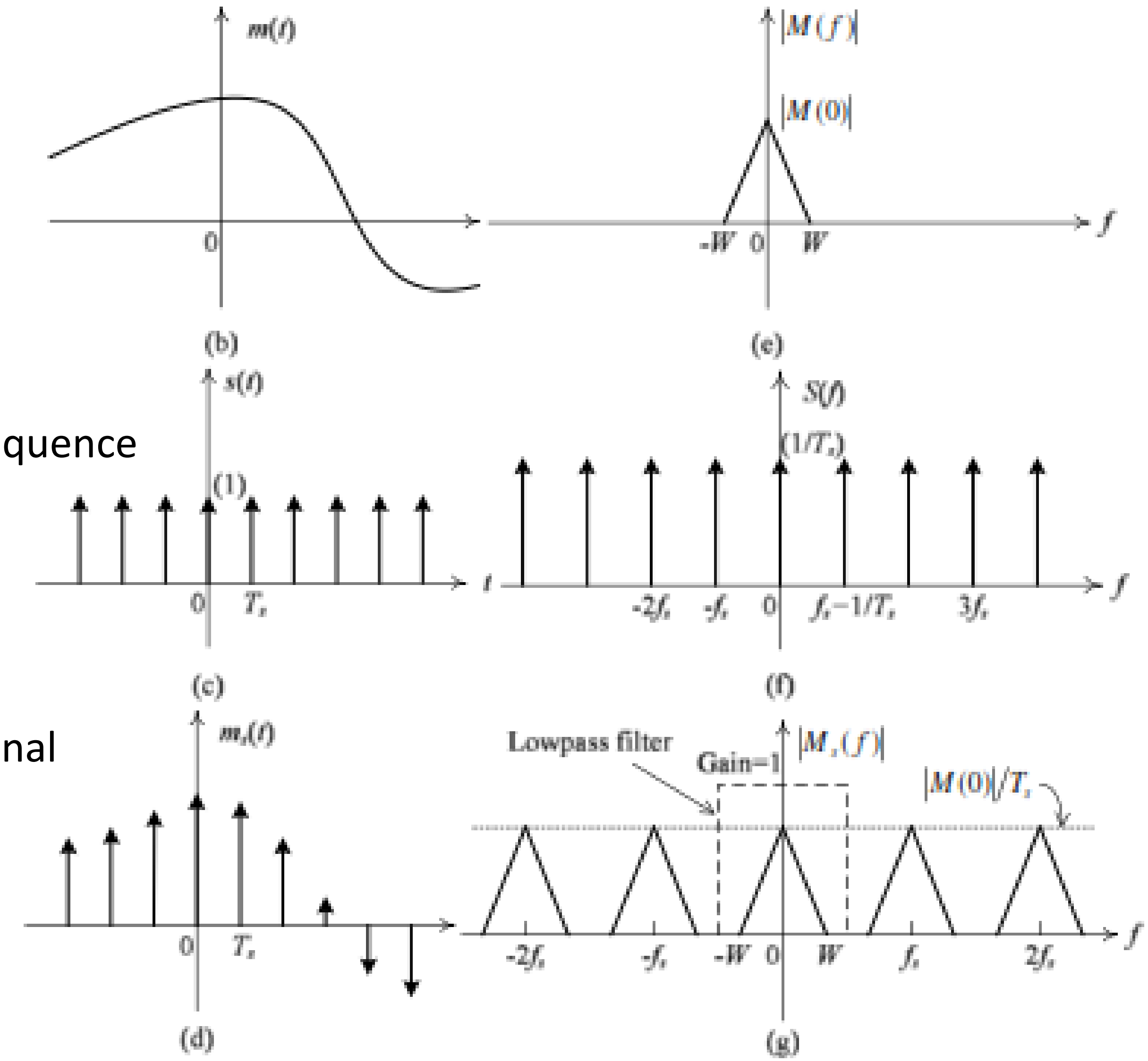
Sample and hold

The Sampling Theorem

message

Sampling Sequence

Sampled Signal



The Sampling Theorem

The Sampling Theorem:

A bandlimited signal with no spectral components above W H_z can be recovered uniquely from its samples taken every T_s seconds, provided that

$$T_s \leq \frac{1}{2W}, \quad \text{or, equivalently,} \quad f_s \geq 2W.$$

Nyquist Rate

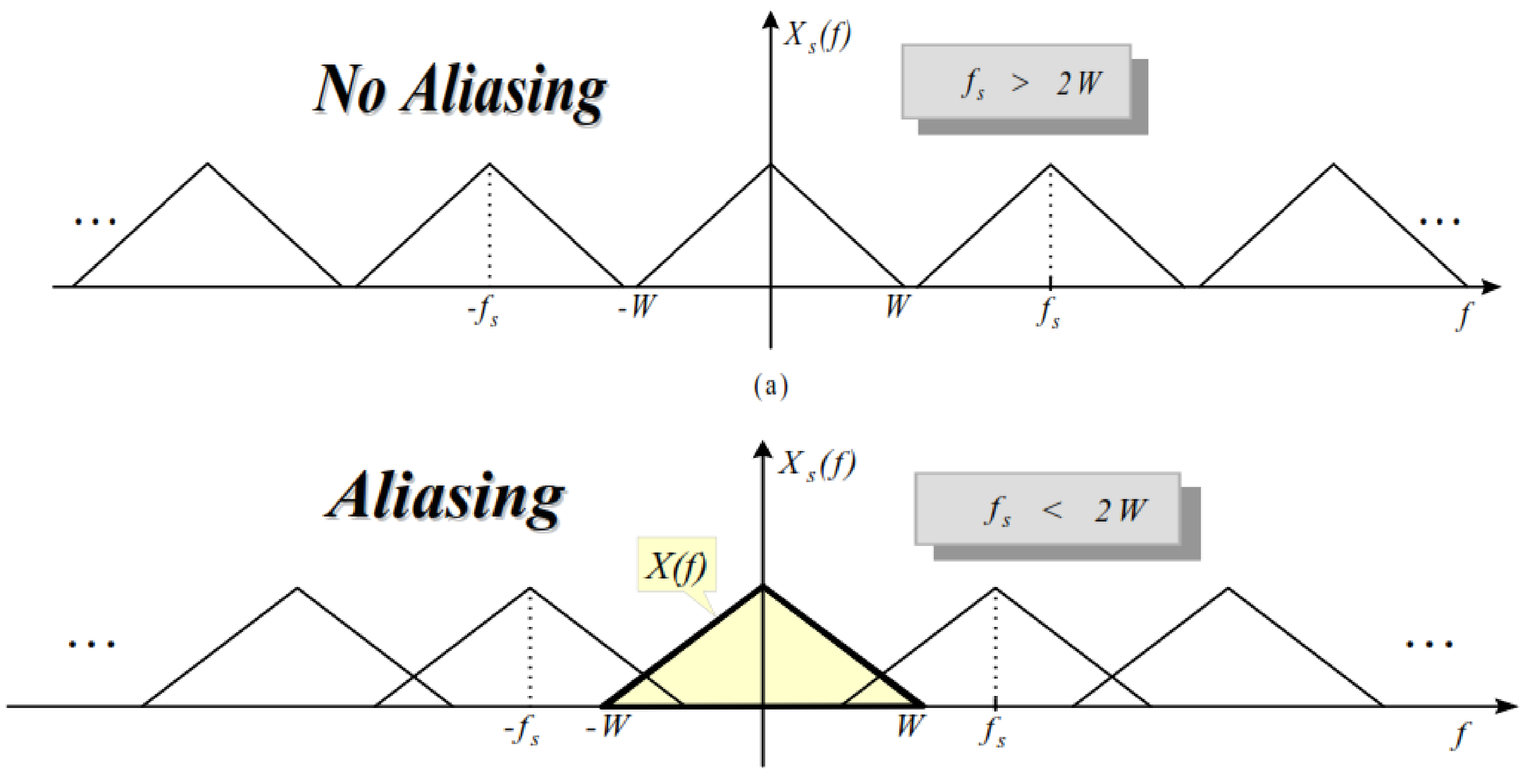
Extraction of $x(t)$ from its samples can be done by passing the sampled signal through a low-pass filter. Mathematically, $x(t)$ can be expressed in terms of its samples by:

$$x(t) = \sum_k x(kT_s) \cdot g(t - kT_s)$$

- The Sampling frequency fs = 2W, is called the Nyquist rate.
- It represents the minimum rate at which a signal must be sampled in order to reconstruct it from its samples without distortion.
- When the sampling rate is less than the Nyquist rate, a distortion type of noise called **Aliasing** results.

Aliasing

- When the sampling frequency is less than the Nyquist rate, aliasing results and the message signal cannot be recovered from the sampled signal without distortion.



Sampling Techniques: Natural Sampling

The band-limited signal x(t) with bandwidth W Hz is multiplied by a periodic sequence of rectangular pulses, $g_p(t)$, with a duty cycle (τ/T_s)

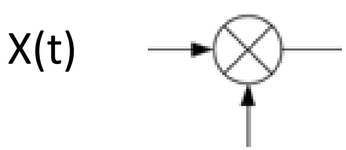
$$x_s(t) = x(t)g_p(t)$$

Expanding $g_p(t)$ in Fourier series, we get

$$x_s(t) = x(t)[C_0 + 2C_1\cos(2\pi f_0 t) + 2C_2\cos(2\pi(2f_0)t) + 2C_3\cos(2\pi(3f_0)t) + \cdots]$$

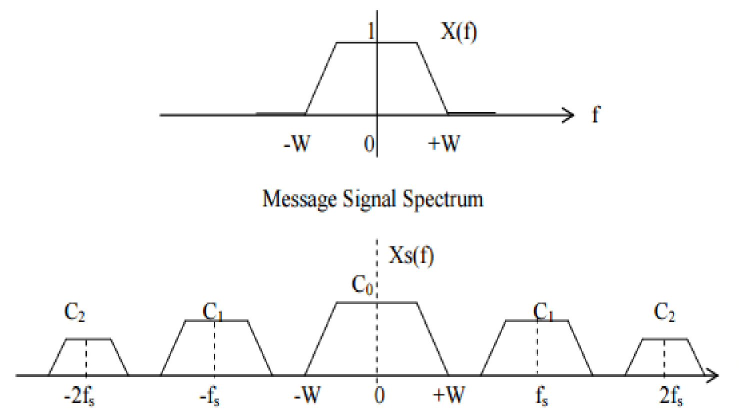
Taking the Fourier transform, and simplifying, we get:

$$X_s(f) = C_0X(f) + C_1X(f - f_s) + C_1X(f + f_s) + C_2X(f - 2f_s) + C_2X(f + 2f_s) + \cdots$$



$$p(t) = \sum_{n=-\infty}^{\infty} h(t - nT_s)$$

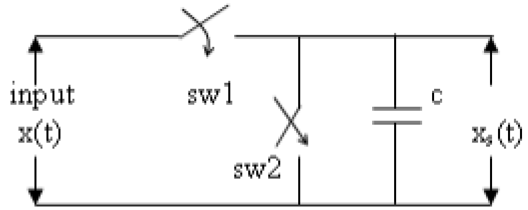
The periodic square function



Message Signal Spectrum

Sampled Signal Spectrum ($f_s > 2W$)

Flat Topped Sampling (Zero order hold sampling)

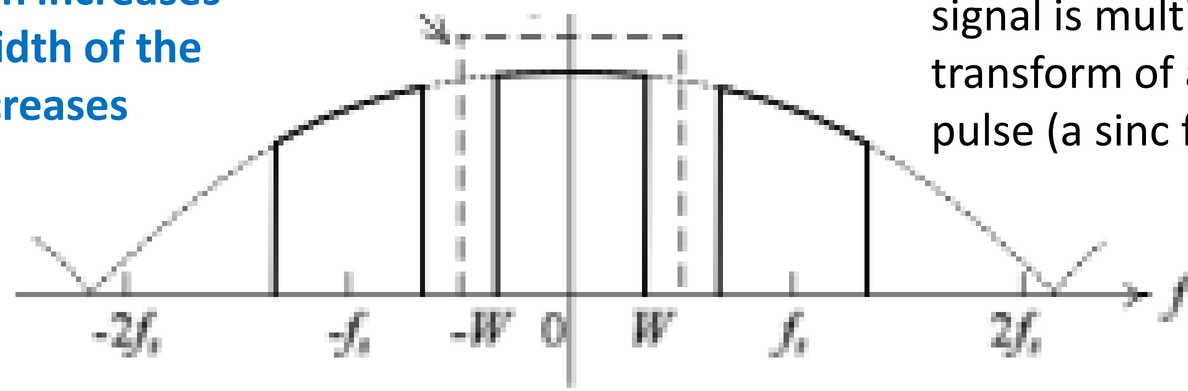


$$x_s(t) = \sum_{-\infty}^{\infty} x(kT_s)p(t - kT_s)$$

$$X_s(f) = \frac{P(f)}{T_s}\sum_{-\infty}^{\infty} X(f - kf_s)$$

$$P(f) = A\tau\,\mathrm{sinc}(f\tau)$$

Distortion increases as the width of the pulse increases



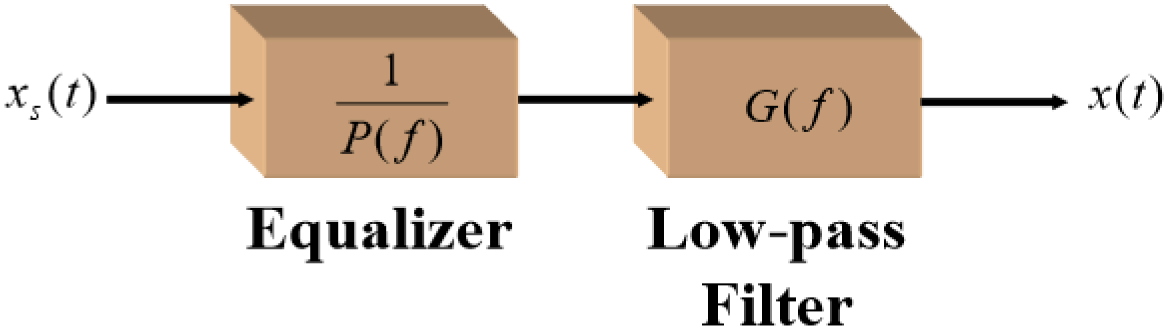
Effect: As if the ideal sampled signal is multiplied by the transform of a rectangular pulse (a sinc function).

Even if the sampling rate>Nyquist rate, distortion results due to the effect of holding the sample for some time τ. We will study the effect of the holding time on the distortion.

Flat Topped Sampling (Zero order hold sampling)

- A distortion-free signal can be obtained by using an equalizing filter whose transfer function is the reciprocal of that to the unit pulse $H_E(f) = 1/P(f)$

$$P(f) = A\tau \,\mathrm{sinc}(f\tau)$$

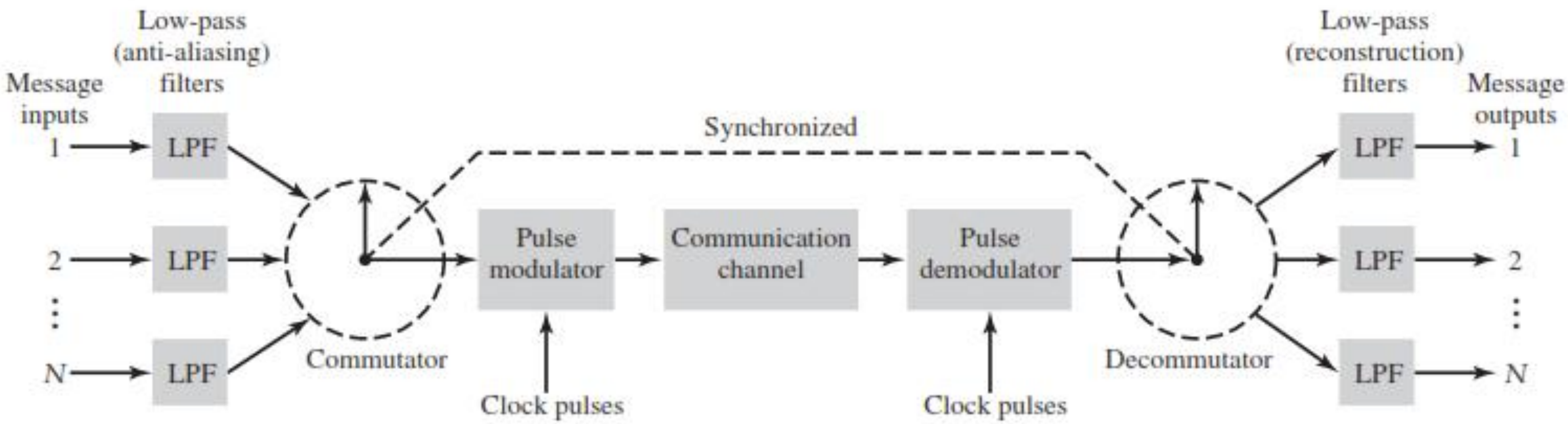


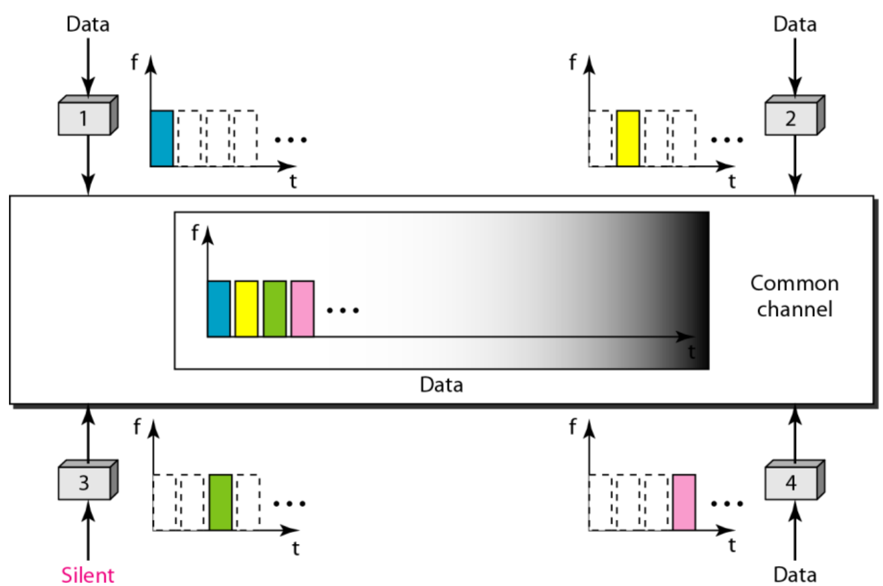
Reconstruction is possible but an equalizer may be needed

Time Division Multiplexing (TDM)

- Let N be the number of sources. The time axis is divided into N slots and each slot is allocated to a source.
- Each source transmits only during its slot, avoiding the possibility of a collision.
- When a user transmits during its slot, it utilizes the entire B.W. of the channel and this B.W. will be made available to the next user during the succeeding time slot.
- The collection of the N slots is called a **cycle**.
- TDMA requires some form of synchronization.
- The number of signal samples transmitted per second should be larger than the Nyquist rate.
- **In this experiment we will multiplex and demultiplex two signals, a triangular and a sine function and observe the multiplexed signal and the demodulated signals.**

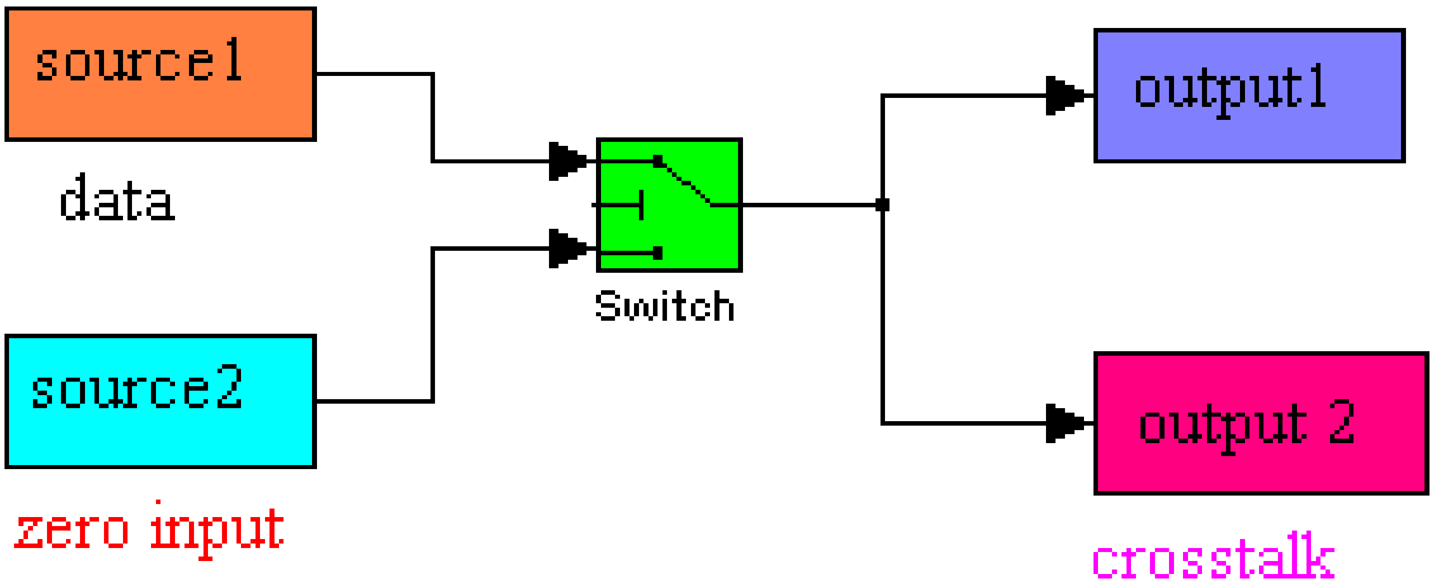
Time Division Multiplexing





Cross Talk

– Due to the limited channel bandwidth and synchronization issues, part of the information transmitted by each user may spill over into the time slot allocated to second user, resulting in a type of noise called cross talk.



You should observe and measure the cross talk in the lab for two users. You will notice the effect of mis-synchronization on the demodulated signals.